COMPARING PREDICTIVE ACCURACY

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True value: $\left\{ y_t \right\}_{t=1}^{T}$

Consider two forecasts: $\left\{ \hat{y}_{it} \right\}_{t=1}^{T}$ and $\left\{ \hat{y}_{jt} \right\}_{t=1}^{T}$

forecast errors $\left\{ e_{it} \right\}_{t=1}^{T}$ and $\left\{ e_{jt} \right\}_{t=1}^{T}$

Loss function $g(y_{it}, \hat{y}_{it}) = g(e_{it})$

$$H_0: \quad E\left[g(e_{it}) \right] = E\left[g(e_{jt}) \right]$$

AN ASYMPTOTIC TEST

Define loss differential: $d_t \triangleq g(e_{it}) - g(e_{jt})$

$$H_0: \quad E\left[d_t\right] = 0$$

$$\bar{d} = \frac{1}{T}\sum_{i=1}^{T} d_t = \frac{1}{T}\sum_{i=1}^{T}\left[g(e_{it}) - g(e_{jt})\right]$$

$$\bar{d} \sim N(\mu, \frac{2\pi f_d(0)}{T})$$

Where $f_d(0) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\gamma_d(\tau)$ **and** $\gamma_d(\tau) = E[(d_t - \mu)(d_{t-\tau} - \mu)]$

AN ASYMPTOTIC TEST

Under H_0 **when** $T \to \infty$

$$S = \frac{\bar{d}}{\sqrt{\frac{2\pi f_d(0)}{T}}} \overset{a}{\sim} N(0,1)$$

Further more $$S_1 = \frac{\bar{d}}{\sqrt{\frac{2\pi \hat{f}_d(0)}{T}}} \overset{a}{\sim} N(0,1)$$

Where $$2\pi\widehat{f_d}(0) = \sum_{\tau=-(T-1)}^{(T-1)} 1\left(\frac{\tau}{S(T)}\right)\widehat{\gamma}_d(\tau),$$ **and**

$$\widehat{\gamma}_d(\tau) = \frac{1}{T}\sum_{t=|\tau|+1}^{T}(d_t - \bar{d})(d_{t-|\tau|} - \bar{d}),$$

AN ASYMPTOTIC TEST

Condition :

loss-differential series is covariance stationary and short memory

Loss function can be asymmetric

forecast errors can be non-Gaussian, nonzero mean, serially correlated, and contemporaneously correlated.

EXACT FINITE-SAMPLE TESTS

When the sample size is small

$$H_0: \quad med\left(g(e_{it}) - g(e_{jt})\right) = 0$$

The sign test:

Test statistic

$$S_2 = \sum_{t=1}^{T} I_+(d_t),$$

where

$$I_+(d_t) = 1 \quad \text{if } d_t > 0$$
$$= 0 \quad \text{otherwise.}$$

Under H_0 $S_2 \sim B(T, 0.5)$ **Further more when** $T \to \infty$

$$S_{2a} = \frac{S_2 - .5T}{\sqrt{.25T}} \overset{a}{\sim} N(0, 1).$$

EXACT FINITE-SAMPLE TESTS

Wilcoxon's *Signed-Rank Test.*

Test statistic $$S_3 = \sum_{t=1}^{T} I_+(d_t)\,\mathrm{rank}(|d_t|),$$

Under H_0 **the distribution of** S_3 **is known** $P(S_3 = t)$ **can be found.**

when $T \to \infty$

$$S_{3a} = \frac{S_3 - \frac{T(T+1)}{4}}{\sqrt{\frac{T(T+1)(2T+1)}{24}}} \overset{a}{\sim} N(0, 1).$$

EXACT FINITE-SAMPLE TESTS

condition

loss-differential series is iid

zero-mean and symmetric the null hypothesis is equal